INTELLIGENT DATA SAMPLING PROMOTES ACCELERATED MEDICAL IMAGING: SHARPER POSITRON EMISSION TOMOGRAPHY

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UMEÅ UNIVERSITY
Positron Emission Tomography

Time in scanner
Radioactive dosage
Machine composition
Noise

Intelligent data sampling
2. Why?

Positron Emission Tomography

| Time in scanner | Radioactive dosage | Machine composition | Noise |

Intelligent data sampling
Positron Emission Tomography

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Cost reduction

More available systems

Intelligent data sampling
Positron Emission Tomography

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Intelligent data sampling
Inspiration

Figure: Left: Full standard 3D Magnetic Resonance Imaging headscan. Middle: Zoom of lower left area. Right: Structured compressed sensing approaches to resolution enhancing.¹

¹Figures from ”Undersampling improves fidelity of physical imaging and the benefits grow with resolution”, B. Roman, R. Calderbank, B. Adcock D. Nietlispach, M. Bostock, I. Calvo-Almazn, M. Graves A. Hansen, PNAS (in revision).
Compressed Sensing

Solve underdetermined linear systems.

\[
\text{[measurements]} = \text{[sensing matrix]} \times \text{[signal]}
\]

Two types of compressed sensing problems:

I. Physical devices impose the sampling operator.

II. Sensing mechanism offers freedom to design the sampling operator.
Compressed Sensing

Solve underdetermined linear systems.

\[
[\text{measurements}] = [\text{sensing matrix}] \times [\text{image}]
\]

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Compressed Sensing

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sparse?

Two types of compressed sensing problems:

I. Physical devices impose the sampling operator.

II. Sensing mechanism offers freedom to design the sampling operator.
Wavelets

3. How?

Wavelets

Haar
Shannon or Sinc
Daubechies 4
Daubechies 20

Gaussian or Spline
Biorthogonal
Mexican Hat
Coiflet

Figure 1.2–3

Examples of types of wavelets. Note 2 wavelets for the Biorthogonal. The Shannon, Gaussian, and Mexican Hat are “crude” wavelets that are defined by an explicit mathematical expression (and whose wavelet filters are obtained from evaluating that expression at specific points in time). The rest are estimations of a “continuous” wavelet built up from the original filter points.

Jargon Alert: Shifting or sliding is often referred to as “translating” in wavelet terminology.

1.3
The value of Transforms and Examples of Everyday Use

Perhaps the easiest way to understand wavelet transforms is to first look at some transforms and other concepts we are already familiar with.

The purpose of any transform is to make our job easier, not just to see if we can do it. Suppose, for example, you were asked to quickly take the year 1999 and double it. Rather than do direct multiplication you would probably do a home-made “millennial transform” in your head something like 1999 = 2000 – 1. Then after transforming you would multiply by 2 to obtain 4000 – 2.

1.6
Examples using the Continuous Wavelet Transform

Wavelet transforms are exciting because they too are comparisons, but instead of correlating with various stretched, constant frequency sinusoid waves they use smaller or shorter waveforms (“wave–lets”) that can start and stop. In other words, the fast Fourier transform relates the signal to sinusoids while the wavelet transforms relate signals to wavelets. In the real world of digital computers, wavelet transforms relate our discrete, finite (digital) signal to the discrete, finite, wavelet filters.

Fig. 1.6–1 shows us some of the constituent wavelets that have been shifted and stretched (from the mother wavelet) that make up the signal. In other words, we are correlating (comparing) the signal with these various shifted, stretched wavelets. An actual wavelet transform compares many stretched and shifted wavelets (“analysis wavelets”) to the original pulse rather than just these few shown in Figure 1.6–1.

Wavelet transform of image: multiscale representation

- coarse scale – low-resolution components
- fine scale – high-resolution components

\[ \text{[image]} = \text{sum [coefficients]} \times \text{[wavelet functions]} \]

Only few of the coefficients are important.
Sparse representation: keep only the important ones and set the rest to zero.
Unknown image $x_0$.

Sampling equipment samples radon transform $Rx_0$.

$x_0$ may not be sparse itself, but its wavelet transform $\tilde{x} = \Phi_{dwt}x_0$ may be. Subsample $\Omega = \{1, \ldots, N\}$ with $m = |\Omega|$ and solve

$$\min ||z||_1 \text{ subject to } P_\Omega R\Phi_{dwt}^{-1}z = P_\Omega R\Phi_{dwt}^{-1}\tilde{x}.$$ 

Notes:

- Subsampling scheme $\Omega$.
- Minimum number of measurements $m$.
- Radon transform.
- Choice of wavelets.
3. How?  
Possibilities

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²Figures from "Undersampling improves fidelity of physical imaging and the benefits grow with resolution", B. Roman, R. Calderbank, B. Adcock D. Nietlispach, M. Bostock, I. Calvo-Almazn, M. Graves A. Hansen, PNAS (in revision).
Thanks for listening!